EXPLICIT SOLUTIONS OF CERTAIN MULTIDIMENSIONAL INVERSE PROBLEMS FOR SYSTEMS OF PARABOLIC-TYPE EQUATIONS

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Explicit solutions have been found for inverse problems of determining the system coefficients of parabolic equations, where the unknown coefficients are functions of time and space variables. The results of numerical calculations are given.

As we study inverse problems related to the determination of the thermophysical characteristics which are system coefficients of parabolic equations, of marked significance is the ascertaining of such special regimes in which these problems allow of explicit solutions [1, 2]. Similar problems are encountered in determining physical characteristics for processes of heat exchange in porous, nonuniform, and other media of complex structure [3, 4]. Here we analyze several such regimes and write out the explicit solutions of the inverse problems.

I. Let us examine the process described by the system

$$C_k(x,\ t)\frac{\partial T_k}{\partial t} - \mathscr{L}[\lambda_k(x,\ t),\ T_k] - b_k(x,\ t)\sum_{i=1}^{n}\frac{\partial T_k}{\partial x_i} - \bar{b}_k(x,\ t)\sum_{i=1}^{m}\frac{\partial T_k}{\partial y_i} + \sum_{j=1}^{N} a_{kj}(x,\ t)T_j(x,\ y,\ t) = Q_k(x,\ y,\ t),$$
$$k = \overline{1,\ N},\ (x,\ y,\ t)\in\Omega, \tag{1}$$

with the following initial and boundary conditions:

$$T_k(x,\ y,\ t)|_{t=0} = \varphi_k(x,\ y), \tag{2}$$

$$T_k(x,\ y,\ t)|_{\bar{D}_1\times\Gamma_2} = 0,\ T_k(x,\ y,\ t)|_{\bar{D}_2\times\Gamma_1} = f_k(\zeta,\ y,\ t), \tag{3}$$

where $C_k > 0$, $\lambda_k > 0$, b_k, $\bar{b}_k$, Q_k, a_{kj}, φ_k, f_k, k, $j = \overline{1,\ N}$ are continuous functions, ζ is a fixed point on the boundary $D_2 \times \Gamma_1$, the operator $\mathscr{L}[\lambda_k(x,\ t),\ T_k]$ has one of the following expressions: 1) $\lambda_k(x,\ T)\Delta T_k$; 2) $\lambda_k(x,\ t)\Delta_x T_k + \Delta_y T_k$; 3) $\Delta_x T_k + \lambda_k(x,\ t)\Delta_y T_k$; 4) $\nabla_x[\lambda_k(x,\ t)\nabla_x T_k] + \lambda_k(x,\ t)\Delta_y T_k$; 5) $\nabla_x[\lambda_k(x,\ t)\nabla_x T_k] + \Delta_y T_k$.

We are interested in the solution of the inverse problem of determining coefficients (1). For this purpose we will connect to system (1)-(3) one or more of the following conditions:

$$T_k(x,\ y,\ t)|_{y=\eta} = \psi_k(x,\ t),\ \eta\in D_2, \tag{4}$$

$$\left.\frac{\partial T_k}{\partial \nu}\right|_{y=\eta} = g_k(x,\ t),\ \overline{\eta\in D_2}, \tag{5}$$

$$\lambda_k(x,\ t)\left.\frac{\partial T_k}{\partial \nu}\right|_{y=\eta} = q_k(x,\ t),\ \eta\in\bar{D}_2,\ k = \overline{1,\ N}, \tag{6}$$

$$\sum_{k=1}^{N}\lambda_k(x,\ t)\sigma_k(x,\ t)\left.\frac{\partial T_k}{\partial \nu}\right|_{y=\eta} = q_0(x,\ t),\ \eta\in\bar{D}_2, \tag{7}$$

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where $\psi_k(x, t)$, $g_k(x, t)$, $q_k(x, t)$, $\sigma_k(x, t)$, $k = \overline{1, N}$, $q_0(x, t)$ are given continuous functions satisfying the required conditions of conformity.

Problem (1)-(3) with given coefficients and right-hand sides is the boundary-value problem for the system of parabolic equations (1). Let us assume that this problem has a single classical solution [5]. We will take w(y) to denote the normalized eigenfunction of the operator Δ_y, corresponding to the eigenvalue $\mu > 0$, i.e.,

$$-\Delta_y w(y) = \mu w(y),\ w(y)|_{\Gamma_2} = 0,\ y \in D_2. \tag{8}$$

Let us examine the heat regime in which the following assumptions (hereafter referred to as assumption A) are realized: $Q_k(x, y, t) = Q_{0k}(x, t) w(y)$, $\varphi_k(x, y) = \varphi_{0k}(x) w(y)$, $f_k(\zeta, y, t) = f_{0k}(\zeta, t)\cdot w(y)$, where $Q_{0k}(x, t)$, $\varphi_{0k}(x)$, $f_{0k}(\zeta, t)$ are given functions. Equations (1) are multiplied by w(y), and then we integrate the derived expressions over the region D_2. If we denote

$$T_{0k}(x, t) = \int_{D_2} T_k(x, y, t) w(y) dy,\ k = \overline{1, N}, \tag{9}$$

then we will obtain

$$C_k(x, t)\frac{\partial T_{0k}}{\partial t} - \mathscr{L}_0[\lambda_k(x, t), T_{0k}] - b_k(x, t)\nabla_x T_{0k} + \sum_{j=1}^{N} a_{kj}(x, t) T_{0k}(x, t) = Q_{0k}(x, t),\ k = \overline{1, N},\ (x, t) \in \Omega_1, \tag{10}$$

where $\mathscr{L}_0[\lambda_k(x, T), T_{0k}]$ is one of the following expressions: 1) $\lambda_k(x, t)[\Delta_x T_{0k} - \mu T_{0k}]$; 2) $\lambda_k(x, t)\cdot \Delta_x T_{0k} - \mu T_{0k}$; 3) $\Delta_x T_{0k} - \mu\lambda_k(x, t) T_{0k}$; 4) $\nabla_x[\lambda_k(x, t)\nabla_x T_{0k}] - \mu\lambda_k(x, t) T_{0k}$; 5) $\nabla_x[\lambda_k(x, t)\nabla_x T_{0k}] - \mu T_{0k}$.

After replacement of (9), conditions (2)-(7) assume the form

$$T_{0k}(x, t)|_{t=0} = \varphi_{0k}(x),\ x \in \bar{D}_1, \tag{11}$$

$$T_{0k}(x, t)|_{\Gamma_1} = f_{0k}(\zeta, t),\ (\zeta, t) \in \Gamma_1, \tag{12}$$

$$T_{0k}(x, t) w(\eta) = \psi_k(x, t),\ \eta \in D_2, \tag{13}$$

$$T_{0k}(x, t)\frac{\partial w}{\partial \nu}(\eta) = g_k(x, t),\ \eta \in \bar{D}_2, \tag{14}$$

$$\lambda_k(x, t) T_{0k}(x, t)\frac{\partial w}{\partial \nu}(\eta) = q_k(x, t),\ \eta \in \bar{D}_2,\ k = \overline{1, N}, \tag{15}$$

$$\sum_{k=1}^{N} \lambda_k(x, t)\sigma_k(x, t) T_{0k}(x, t)\frac{\partial w}{\partial \nu}(\eta) = q_0(x, t),\ \eta \in \bar{D}_2. \tag{16}$$

Thus, under assumption A system (1)-(7) is equivalently transformed to system (10)-(16). Therefore, in the following we will basically consider system (10)-(16).

Let $C_k(x, t) > 0$, $b_k(x, t)$, $\bar{b}_k(x, t)$, $a_{kj}(x, t)$, $k, j = \overline{1, N}$ be continuous functions given on Ω_1 and from conditions (1)-(4) we have to determine the continuous and positive functions $\lambda_k(x, t)$, $k = \overline{1, N}$. If we take (13) into consideration in (10), we will obtain

$$-\mathscr{L}_0[\lambda_k(x, t), \psi_k] = \Phi_k(x, t),\ (x, t) \in \Omega_1,\ k = \overline{1, N}, \tag{17}$$

where

$$\Phi_k(x, t) = b_k(x, t)\nabla_x\psi_k - \sum_{j=1}^{N} a_{kj}(x, t)\psi_k - C_k(x, t)\frac{\partial \psi_k}{\partial t} + Q_{0k}(x, t) w(\eta).$$

In the case of forms 1)-3) of the operator $\mathscr{L}_0[\lambda_k(x, t), T_{0k}]$, the function $\lambda_k(x, t)$ from (17) is determined elementarily. However, if the operator $\mathscr{L}_0[\lambda_k(x, t), T_{0k}]$ has the form of 4) and 5), then for the determination of $\lambda_k(x, t)$ we derive a system of first-order differential equations from which $\lambda_k(x, t)$ can be found with additional assumptions. For example, in the case of form 4) of the operator $\mathscr{L}_0$ with $n = 1$, $\bar{D}_1 = [0, 1]$, let us assume that the functions $\partial\psi_k(x, t)/\partial x$ vanish only at the point $x_0 \in [0, 1]$. Then from (17) we obtain

$$\lambda_k(x_0, t) = \Phi_k(x_0, t)\left[\mu\psi_k(x_0, t) - \frac{\partial^2\psi_k(x_0, t)}{\partial x^2}\right]^{-1}, \quad k = \overline{1, N}. \tag{18}$$

Let us now solve system (17) under conditions (18). We know that this solution has the form [6]

$$\lambda_k(x, t) = \exp\left\{-\int_{x_0}^{x} P_k(z)\,dz\right\}\left[\int_{x_0}^{x} R_k(z)\exp\left\{\int_{x_0}^{z} P_k(s)\,ds\right\}dz + \lambda_k(x_0, t)\right], \tag{19}$$

where

$$P_k(z) = \left[\frac{\partial^2\psi_k(z, t)}{\partial z^2} - \mu\psi_k(z, t)\right]\left[\frac{\partial\psi_k(z, t)}{\partial z}\right]^{-1},$$

$$R_k(z) = \Phi_k(z, t)\left[\frac{\partial\psi_k(z, t)}{\partial z}\right]^{-1}, \quad z \neq x_0, \quad k = \overline{1, N}.$$

The right-hand sides of expressions (18) and (19) are assumed to be positive, continuous, and finite.

If the coefficients $\lambda_k(x, t)$, $k = \overline{1, N}$ are given and we are seeking the coefficients $C_k(x, t) > 0$, $b_k(x, t)$ or $a_{k\ell}(x, t)$, $k = \overline{1, N}$, then they can also be found from conditions (1)-(4) with the method described above. Here ℓ is one of the numbers 1, 2, ..., N. For example, if $\lambda_k(x, t) > 0$, $b_k(x, t)$, $a_{kj}(x, t)$, $k, j = \overline{1, N}$ are given functions, then for $C_k(x, t)$ the following expressions are valid:

$$C_k(x, t) = \left\{Q_{0k}(x, t)w(\eta) + \mathscr{L}_0[\lambda_k(x, t), \psi_k] + b_k(x, t)\nabla_x\psi_k(x, t) - \right.$$
$$\left. - \sum_{j=1}^{N} a_{kj}(x, t)\psi_k(x, t)\right\}\left[\frac{\partial\psi_k(x, t)}{\partial t}\right]^{-1}, \quad k = \overline{1, N}.$$

The right-hand sides of these expressions are assumed to be positive, continuous, and finite. If in the above-examined problems, in the place of (4) we adopt condition (5), then in the above-derived explicit formulas for the unknown coefficients $w(\eta)$, $\psi_k(x, t)$ should be replaced by $(\partial w/\partial\nu)(\eta)$ and $g_k(x, t)$, respectively.

Now let us consider the problem of determining the functions $\lambda_k(x, t)$, $k = \overline{1, N}$ from system (1)-(3), (6). It is assumed that the other coefficients of the equation are given. After substitution of (9) system (1)-(3), (6) is transformed to system (10)-(12), (15). From (15) and (10) we have

$$C_k(x, t)\frac{\partial T_{0k}}{\partial t} - \mathscr{L}_0\left[q_k(x, t)\left(T_{0k}\frac{\partial w}{\partial\nu}(\eta)\right)^{-1}, T_{0k}\right] - b_k(x, t)\nabla_x T_{0k} +$$
$$\sum_{j=1}^{N} a_{kj}(x, t)T_{0k} = Q_{0k}(x, t), \quad k = \overline{1, N}, \quad (x, t) \in \Omega_1. \tag{20}$$

Consequently, $T_{0k}(x, t)$ is a solution of the mixed problem for the system of quasilinear parabolic equations (20) with conditions (11) and (12). Let us assume that $q_k(x, t) > 0$, $\varphi_{0k}(x) > 0$, $f_{0k}(\zeta, t) > 0$, $\partial w/\partial\nu(\eta) > 0$). These assumptions can be realized in practical terms, and a portion of these conditions is associated with the "conformity" of the initial data. Then the function $T_{0k}(x, t) > 0$, and from (20), (11), and (12) they can be found exactly or approximately. Substituting the found expressions for the functions $T_{0k}(x, t)$ into (15), we will obtain

$$\lambda_k(x, t) = q_k(x, t)\left[T_{0k}(x, t)\frac{\partial w}{\partial \nu}(\eta)\right]^{-1}, \ k=\overline{1, N}. \tag{21}$$

If the additional conditions are specified in the form of (7), then the method described above for the unknown coefficients yields expressions analogous to (19) and (21).

Let us present one special case of an inverse problem such as we considered above, which is encountered in studying the processes of cooling a body by a flow of liquid or gas, as the flow changes speed or temperature. Suppose we have to find a function $a(t)$ continuous on $[0, t_{ob}]$ from the system

$$\frac{\partial T_i}{\partial t} - \Delta_x T_i + (-1)^i a(t)(T_2 - T_1) = 0, \quad (x, t)\in\Omega_1, \ i=1, 2, \tag{22}$$

with the initial and boundary conditions

$$T_i|_{t=0} = \varphi_i(x), \quad T_i|_{\Gamma_1} = 0, \quad i=1, 2, \ x\in\overline{D}_1, \tag{23}$$

and the following additional condition:

$$\frac{\partial T_2}{\partial \nu}(\eta, t) = g(t), \quad 0\leqslant t\leqslant t_{ob}, \tag{24}$$

where $\varphi_i(x)$ and $g(t)$ are given functions.

Let D_1 be a region such that the Green's function $G_1(x, t; \zeta, \theta)$ of the first boundary-value problem for the equation $Yt - \Delta Y = 0$ can be found in explicit form. For example, D_1 may be a half space, a sphere, a segment, etc. In system (22)-(24), having carried out the substitution

$$v_i(x, t) = T_i(x, t)\exp\left\{\int_0^t a(\tau)\,d\tau\right\}, \quad i=1, 2,$$

after uncomplicated transformations for the unknown coefficient we obtain the expression

$$a(t) = \frac{1}{2}\frac{d}{dt}\ln\left\{\frac{\int_{D_2}\frac{\partial G_1}{\partial \nu}(\eta, t; \zeta, 0)[\varphi_2(\zeta)-\varphi_1(\zeta)]\,d\zeta}{\int_{D_2}\frac{\partial G_1}{\partial \nu}(\eta, t; \zeta, 0)[\varphi_1(\zeta)+\varphi_2(\zeta)]\,d\zeta - 2g(t)}\right\}.$$

If in problem (22)-(24) in the place of conditions $T_i|_{\Gamma_1} = 0$ we specify the conditions $(\partial T_i/\partial\nu)|_{\Gamma_1} = 0$, then in the place of (24), setting the condition

$$T_2(\eta, t) = \psi(t), \quad 0\leqslant t\leqslant t_{ob},$$

for the unknown coefficient $a(t)$ we will find the expression

$$a(t) = \frac{1}{2}\frac{d}{dt}\ln\left\{\frac{\int_{D_2} G_2(\eta, t; \zeta, 0)[\varphi_2(\zeta)-\varphi_1(\zeta)]\,d\zeta}{\int_{D_2} G_2(\eta, t; \zeta, 0)[\varphi_2(\zeta)+\varphi_1(\zeta)]\,d\zeta - 2\psi(t)}\right\},$$

where $G_2(x, t; \zeta, \theta)$ is the Green's function of the second boundary-value problem for the equation $Y_t - \Delta Y = 0$.

II. Let us now examine the process described by the following system:

$$C_k(x, t)\frac{\partial T_k}{\partial t} - \mathscr{L}[\lambda_k(x, t), T_k] - b_k(x, t)\sum_{i=1}^{n}\frac{\partial T_k}{\partial x_i} - \overline{b}_k(x, t)\times \tag{25}$$

$$\times\sum_{i=1}^{m}\frac{\partial T_k}{\partial y_i} + a_k(x, t)T_k(x, y, t) = Q_k(x, y, t), \ (x, y, t)\in\Omega_{0k}, \ k=\overline{1, N},$$

$$T_k(x,\ y,\ t)|_{t=0} = \varphi_k(x,\ y),\quad (x,\ y)\in \bar{D}_{1k}\times\bar{D}_2,\quad k=\overline{1,\ N},$$
$$T_k|_{\bar{D}_{1k}\times\Gamma_2} = 0,\ k=\overline{1,\ N},\ T_1|_{\gamma_0} = f_1(\zeta_0,\ y,\ t), \tag{26}$$

$$T_k(x,\ y,\ t)|_{\gamma_k-0} = T_k(x,\ y,\ t)|_{\gamma_k+0},\quad \lambda_k(x,\ t)\frac{\partial T_k}{\partial \nu}\Big|_{\gamma_k-0} =$$
$$\lambda_{k+1}(x,\ t)\frac{\partial T_{k+1}}{\partial \nu}\Big|_{\gamma_k+0},\quad k=\overline{2,\ N-1},\ T_N|_{\gamma_N} = f_N(\zeta_N,\ y,\ t). \tag{27}$$

Problem (25)-(27) with given coefficients and right-hand sides is the boundary-value problem for the system of parabolic equations. The functions $T_k(x,\ y,\ t)$ are a classical solution of this problem. When conditions A are met, after substitution of (9), system (25)-(27) assumes the form

$$C_k(x,\ t)\frac{\partial T_{0k}}{\partial t} - \mathcal{L}_0[\lambda_k(x,\ t),\ T_{0k}] - b_k(x,\ t)\sum_{i=1}^{n}\frac{\partial T_{0k}}{\partial x_i} +$$
$$a_k(x,\ t)T_{0k}(x,\ t) = Q_{0k}(x,\ t),\quad (x,\ t)\in\Omega_{1k},\quad k=\overline{1,\ N}, \tag{28}$$

$$T_{0k}(x,\ t)|_{t=0} = \varphi_{0k}(x),\quad x\in\bar{D}_{1k},\quad k=\overline{1,\ N}, \tag{29}$$

$$T_{01}|_{\gamma_0} = f_{01}(\zeta_0,\ t),\quad T_{0k}(x,\ t)|_{\gamma_k-0} = T_{0k+1}(x,\ t)|_{\gamma_k+0},$$
$$\lambda_k(x,\ t)\frac{\partial T_{0k}}{\partial\nu}\Big|_{\gamma_k-0} = \lambda_{k+1}(x,\ t)\frac{\partial T_{0k+1}}{\partial\nu}\Big|_{\gamma_k+0}, \tag{30}$$
$$T_{0N}|_{\gamma_N} = f_{0N}(\zeta_N,\ t),\quad k=\overline{2,\ N-1}.$$

Let $C_k(x,\ t) > 0$, $b_k(x,\ t)$, $a_k(x,\ t)$, $k = \overline{1,\ N}$ be continuous functions given on the regions Ω_{1k} and from conditions (25)-(27), (5) we have to determine the continuous and positive functions $\lambda_k(x,\ t)$, $k = \overline{1,\ N}$. Considering (14) in (28), we obtain

$$-\mathcal{L}_0[\lambda_k(x,\ t),\ g_k(x,\ t)] = \Phi_k(x,\ t),\quad k=\overline{1,\ N}, \tag{31}$$

where

$$\Phi_k(x,\ t) = Q_{0k}(x,\ t)\frac{\partial\omega}{\partial\nu}(\eta) - C_k(x,\ t)\frac{\partial g_k(x,\ t)}{\partial t} + b_k(x,\ t)\sum_{i=1}^{n}\frac{\partial g_k(x,\ t)}{\partial x_i} - a_k(x,\ t)g_k.$$

When the operator $\mathcal{L}_0[\lambda_k(x,\ t),\ T_{0k}]$ has the form of 1)-3), the functions $\lambda_k(x,\ t)$ are determined in elementary fashion from (31). If the operator $\mathcal{L}_0[\lambda_k,\ T_{0k}]$ has the form of 4) and 5), then for the determination of $\lambda_k(x,\ t)$ we obtain a system of first-order partial differential equations from which the functions $\lambda_k(x,\ t)$ can be found under additional assumptions. Thus, for example, in the case of form 4) of the operator $\mathcal{L}_0[\lambda_k,\ T_{0k}]$ when $n = 1$, $D_{01} = [0,\ 1]$, without excluding generality, we assume that $\partial g_1(x,\ t)/\partial x$ vanishes only at the point $x_0 \in [0,\ 1]$. Then from (31) we obtain

$$\lambda_1(x_0,\ t) = \Phi_1(x_0,\ t)\left[\mu g_1(x_0,\ t) - \frac{\partial g_1(x_0,\ t)}{\partial x}\right]^{-1}. \tag{32}$$

Having solved Eq. (31) for the case in which $k = 1$, we find that

$$\lambda_k(x,\ t) = \exp\left\{-\int_{x_{k-1}}^{x} P_k(z)\,dz\right\}\left[\int_{x_{k-1}}^{x} R_k(z)\exp\left\{\int_{x_{k-1}}^{z} P_k(s)\,ds\right\}dz + \lambda_k(x_{k-1},\ t)\right], \tag{33}$$

where

$$P_k(z) = \left[\frac{\partial^2 g_k(z,\ t)}{\partial z^2} - \mu g_k(z,\ t)\right]\left[\frac{\partial g_k(z,\ t)}{\partial z}\right]^{-1},$$

$$R_k(z) = \Phi_k(z, t)\left[\frac{\partial g_k(z, t)}{\partial z}\right]^{-1}, \quad z \neq x_{k-1}.$$

From condition (30) we obtain

$$\lambda_{k+1}(x, t)|_{x=x_k} = \lambda_k(x, t)\frac{\partial g_k(x, t)}{\partial x}\left[\frac{\partial g_{k+1}(x, t)}{\partial x}\right]^{-1}\Bigg|_{x=x_k}, \tag{34}$$

where x_k, $k = \overline{1, N-1}$ are certain numbers. Expressions (34) are used to find $\lambda_{k+1}(x, t)$ from formula (33), etc., for $k = 2, 3, \ldots, N$. It is assumed that the right-hand sides of expressions (32)-(34) are positive, continuous, and finite.

In addition to the additional condition (5) we can also specify one of the conditions (4) and (6). In all of these cases for $\lambda_k(x, t)$ we obtain explicit expressions. If $\lambda_k(x, t)$ are given, then in analogy with the method described above, we can solve the problem of determining other unknown coefficients.

In conclusion, let us note several other types of boundary conditions which also make it possible for us to obtain explicit solutions of inverse problems for Eqs. (1) and (25). With $b_k(x, t) \equiv \bar{b}_k(x, t) \equiv 0$, instead of conditions $T_k|_{\bar{D}_1\times\Gamma_2} = 0$ and $T_k|_{\bar{D}_{1k}\times\Gamma_2} = 0$, in (3) and (27), respectively, we can specify the conditions

$$\frac{\partial T_k}{\partial \nu}\bigg|_{\bar{D}_1\times\Gamma_2} = 0, \qquad \frac{\partial T_k}{\partial \nu}\bigg|_{\bar{D}_{1k}\times\Gamma_2} = 0. \tag{35}$$

Then $w(y)$ is a solution of the problem

$$-\Delta_y w(y) = \mu w(y), \qquad \frac{\partial w}{\partial \nu}\bigg|_{\Gamma_2} = 0.$$

Here the additional conditions (4) may be set for $\eta \in \bar{D}_2$, while conditions (5)-(7) on $\eta \in D_2$ must be given.

In problem (1)-(3), instead of condition $T_k|_{\bar{D}_2\times\Gamma_1} = f_1(\zeta, y, t)$ we can have one of the following conditions:

$$\text{a)}\ \frac{\partial T_k}{\partial \nu}\bigg|_{\Gamma_1\times\bar{D}_2} = f_k(\zeta, y, t); \quad \text{b)}\ \lambda_k(x, t)\frac{\partial T_k}{\partial \nu}\bigg|_{\Gamma_1\times\bar{D}_2} = f_k(\zeta, y, t).$$

Analogously, in problem (25)-(27), instead of conditions $T_1|_{\gamma_0} = f_1$, $T_N|_{\gamma_N} = f_N$ we can set one of the following conditions:

$$\text{c)}\ \frac{\partial T_1}{\partial \nu}\bigg|_{\gamma_0} = f_1(\zeta_0, y, t), \quad \frac{\partial T_N}{\partial \nu}\bigg|_{\gamma_N} = f_N(\zeta_N, y, t);$$

$$\text{d)}\ \lambda_1(x, t)\frac{\partial T_1}{\partial \nu}\bigg|_{\gamma_0} = f_1(\zeta_0, y, t), \quad \lambda_N(x, t)\frac{\partial T_N}{\partial \nu}\bigg|_{\gamma_N} = f_N(\zeta_N, y, t).$$

In these cases we also obtain explicit expressions for the sought coefficients of Eqs. (1) or (25). The method of solving these problems and the form of the derived expressions for the sought coefficients differ little from the methods examined above. Therefore, we will not dwell in detail on all of these problems. We will present only one of them. Let it be required to determine the coefficients $\lambda_k(x, t) > 0$, $k = \overline{1, N}$ from conditions (1), (2), (4), (35) and b). It is assumed in this case that $b_k(x, t) = b_k(x, t) = 0$, $C_k(x, t) > 0$, $a_{kj}(x, t)$ are given functions. Following the same considerations as in the derivation of Eq. (17) for cases of forms 4) and 5) of the operator $\mathscr{L}_0[\lambda_k, T_{0k}]$, we find that $\lambda_k(x, t)$ satisfy the system of partial differential equations (17). Let us connect the following conditions to system (17):

TABLE 1. Comparison of Exact and Approximate Values for the Coefficient $\lambda_1(x, t)$

$\lambda_1(x, t)$		x									
		0	0,11	0,22	0,33	0,44	0,56	0,67	0,78	0,89	1,0
I	Exact	1,6	1,7	1,8	1,9	2,0	2,1	2,2	2,3	2,4	2,5
II	$\delta=0$	1,6	1,661	1,741	1,831	1,926	2,028	2,142	2,288	2,495	2,763
III	$\delta=0,05$	1,6	1,696	1,815	1,947	2,089	2,230	2,400	2,586	2,802	3,000
IV	$\delta=0,15$	1,6	1,769	1,971	2,200	2,450	2,714	2,987	3,255	3,468	3,486

$$\lambda_k(x, t)|_{\bar{D}_2\times\Gamma_1} = f_{0k}(\zeta, t)\left[\frac{\partial\psi_k}{\partial\nu}\Big|_{\bar{D}_2\times\Gamma_1}\right]^{-1} w(\eta), \quad \eta\in\Gamma_2, \tag{36}$$

and these are obtained from b) with consideration of (9) and (13). If n = 1 and the operator $\mathcal{L}_0$ has the form of 4), then solving system (17) under conditions (36), for $\lambda_k(x, t)$ we obtain expressions analogous to (19).

The explicit expressions derived in Secs. I and II make it possible for us to develop simple stable algorithms for the approximate solution of the multidimensional inverse problems examined above [7].

<u>Example.</u> Let $\bar{D}_1 = [0, 1]$, $\bar{D}_2 = [0, 2]$. We have to determine the coefficients $\lambda_1(x, t) > 0$, $\lambda_2(x, t) > 0$ from conditions

$$C_k(x, t)\frac{\partial T_k}{\partial t} - \nabla[\lambda_k(x, t)\nabla T_k] - (-1)^k a(x, t)(T_2 - T_1) = 0, \quad k=1, 2, \tag{37}$$

$$T_k|_{t=0} = \varphi_{0k}(x)\sin 2\pi y, \quad T_k|_{x=0} = f_{1k}(t)\sin 2\pi y, \quad T_k|_{x=1} = f_{2k}(t)\sin 2\pi y, \tag{38}$$

$$\begin{gathered} T_k|_{y=0} = T_k|_{y=2} = 0, \quad \left(\lambda_1\frac{\partial T_1}{\partial y} + \lambda_2\frac{\partial T_2}{\partial y}\right)\Big|_{y=0} = q_0(x, t), \\ \lambda_2\frac{\partial T_2}{\partial y}\Big|_{y=2} = q_1(x, t), \end{gathered} \tag{39}$$

where

$$\begin{gathered} C_1(x, t) = 4\pi^2(x+t+1); \quad C_2(x, t) = (4\pi^2 - 1,5)(x+t+2)+2; \\ a(x, t) = x+t; \quad \varphi_{0k}(x) = k\exp\{-x\}; \quad f_{1k}(t) = k\exp\{-t\}; \\ f_{2k}(t) = k\exp\{-1-t\}, \ k=1, 2; \quad q_0(x, t) = \pi\exp\{-x-t\}\times \\ \times(6x+6t+10); \quad q_1(x, t) = 4\pi\exp\{-x-t\}(x+t+2). \end{gathered}$$

The exact solution of this problem is $\lambda_1(x, t) = x + t + 1$, $\lambda_2(x, t) = x + t + 2$, $T_k(x, y, t) = k\exp\{-x - t\}\sin 2\pi y$, k = 1, 2. It follows from (6) and (7) that

$$\begin{gathered} \lambda_1(x, t) = [q_0(x, t) - q_1(x, t)][2\pi T_{01}(x, t)]^{-1}, \\ \lambda_2(x, t) = q_1(x, t)[2\pi T_{02}(x, t)]^{-1}, \end{gathered} \tag{40}$$

where $T_{0k}(x, t)$ satisfies the conditions of the system which are obtained from (37)-(39) by transformation of (9). In this case $w(y) = \sin 2\pi y$. Solving this problem by the grid method [8], we determine $T_{0k}(x, t)$, k = 1, 2. Subsequently, using (40), we find the values of the coefficients $\lambda_1(x, t)$, $\lambda_2(x, t)$.

For purposes of comparison, Table 1 shows the exact and approximate values of the coefficient $\lambda_1(x, t)$ for t = 0.6 at the nodes $\{x: x = x_i,\ x_i = ih,\ h = 1/9,\ i = 0, 1, \ldots, 9\}$ of the grid. In row I we have the exact values, while rows II-IV give the approximate values of $\lambda_1(x, t)$ for various errors δ in the initial data.

NOTATION

t, time; t_{ob}, observation time interval; D_1, D_2, bounded regions in n-dimensional and m-dimensional Euclidean spaces E_n, E_m; Γ_1, Γ_2, boundaries; $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_m)$, arbitrary points in the regions D_1, D_2; D_{1k}, $k = \overline{1, N}$ are bounded regions in the n-dimensional Euclidean space; $D_1 = \bigcup_{k=1}^{N} D_{1k}$; γ_k, the discontinuity lines of the first kind for region D_1; the signs +0 and −0 indicate the corresponding limits to the right and to the left of the inside normal; T_k, temperature distribution; C_k, volumetric heat capacity; λ_k, coefficient of thermal conductivity; b_k, coefficient characterizing the mobility of the medium; a_k, heat-exchange factor; Q_k, the strength of the inner sources; φ_k, f_k, temperature distributions at the beginning of the process and at the boundaries of the region; ν, external conormal of the boundary; ψ_k, temperature distribution at a given point; g_k, temperature gradient; q_k, density of the heat flow; q_0, density of the total flow; η, ζ, ζ_0, ζ_N, fixed points;

$D \equiv D_1 \times D_2$, $\Omega \equiv D \times (0, t_{ob}]$, $\Omega_1 = D_1 \times (0, t_{ob}]$, $\Omega_2 = D_2 \times (0, t_{ob}]$, $\Omega_{0k} \equiv D_{1k} \times D_2 \times (0, t_{ob}]$, $\Omega_{1k} \equiv D_{1k} \times (0, t_{ob}]$, $\Delta_{x_k} \equiv \frac{\partial^2}{\partial x_k^2}$, $\Delta_{y_k} \equiv \frac{\partial^2}{\partial y_k^2}$, $\nabla_{x_k} \equiv \frac{\partial}{\partial x_k}$, $\nabla_{y_k} \equiv \frac{\partial}{\partial y_k}$, $\Delta_x \equiv \sum_{k=1}^{n} \Delta_{x_k}$, $\Delta_y \equiv \sum_{k=1}^{m} \Delta_{y_k}$, $\Delta \equiv \Delta_x + \Delta_y$, $\nabla x \equiv (\nabla_{x_1}, \ldots, \Delta_{x_n})$, $\nabla y \equiv (\nabla_{y_1}, \ldots, \nabla_{y_m})$.

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